THE MEASUREMENT OF THE STRENGTH OF A MAGNETIC FIELD BY MEANS OF LIQUIDS.

by

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A thesis submitted to the Department of Physics and the Faculty of the Graduate School in partial fulfillment of the requirements for the Master's degree.

Approved: F. E. Kester.

July 26, 1915.

CONTENTS.

ABBREVIATIONS USED.

a, β, c	components of induced magnetization.
α, β, γ	" " field.
a, b, c	" " induction.
χ	susceptibility.
μ	permeability.
F	force.
H	field strength.
g	acceleration due to gravity.
I	current strength.
M	mutual inductance.
I_m	current strength in the magnet circuit.
I_p	" " " " primary " .
$G\theta_p$	galvanometer throw for calibrating.
$G\theta_H$	" " " field strength.
A	effective area.
σ	density.
k	galvanometer constant.

NOTES.

(1) Michael Faraday .... Exp. Researches in Electricity .. pp. 88.

(2) J.H. Jeans ... Electricity and Magnetism .. Chap.XII.

(3) G. Quincke ... Wied.Ann. . 24, 347, 1885.

(4) J.A. Fleming and James Dewar ..... Proc. Roy. Soc. 63, 311, 1898.

(5) Herman D. Stearns.. Phys. Rev. 16, 1, 1908.

(6) H.C. Hayes ..... " " .... April, 1914.

(7) G. Jager and St. Meyer .. Ann. d. Phys. 67, 427, 1899.

(8) A.P. Wills ..... Phil. Mag. (5) 45, 438, 1898.

(9) S.C. Laws ..... " " (6) 8, 49, 1904.

(10) O. Liebknecht u A.P. Wills .. Ann. d. Phys.(4) 1, 181, 1900.

(11) J.A. Fleming and James Dewar .. Proc. Roy. Soc. 60, 283, 1897.

THE MEASUREMENT OF THE STRENGTH OF A MAGNETIC FIELD BY MEANS OF LIQUIDS.

After considering several problems as objects of investigation, this problem as suggested by Mr. T.T. Smith of the Physics Department of making use of the magnetic susceptibility of soluble salts for the measurement of the strengths of magnetic fields was chosen.

I wish to thank the members of the Physics Department for their generosity in providing the necessary apparatus, and especially thank Dr. F.E. Kester for his kind assistance and supervision and Professor H.P. Cady for his suggestion of using the two liquids in a continuous container.

Theory It was Faraday's (1) idea that all substances were either magnetic or diamagnetic, and that any substance placed in a magnetic field would respond, if only to a slight degree, either as a magnetic or diamagnetic body. He admitted the possibility that some might be found to lie at the zero point, but so far no absolutely null substance has been found.

The theory of the action of a magnetic field upon a body placed in it is very clearly discussed by J.H. Jeans (2) whose argument is partly reproduced here. For the purposes

of this paper only non-crystalline substances are considered, since only liquids are used in this experiment. When such materials are placed in the field the direction of the field H and of the induced magnetization are the same and the components of the induced magnetization A, B, C, and the components of the field α, β, γ bear the following relationship:

$$1. \quad \begin{cases} A = \chi\alpha. \\ B = \chi\beta. \\ C = \chi\gamma. \end{cases}$$

in which the constant χ retains the same value and is called the magnetic susceptibility of the given substance. Or, in other words, the magnetic susceptibility of a given substance is the ratio per unit volume of the intensity of magnetization to the field strength. The components of induction a, b, c, are connected in the following manner with the field strengths and intensity of magnetization:

$$2. \quad \begin{cases} a = \alpha + 4\pi A = (1 + 4\pi\chi)\alpha. \\ b = \beta\ 4\pi B = (1 + 4\pi\chi)\beta. \\ c = \gamma\ 4\pi C = (1 + 4\pi\chi)\gamma. \end{cases}$$

The ratio of field strength to induction is equal to

$$3. \quad 1 + 4\pi\chi.$$

and is called permeability, μ, of the substance. The energy per unit volume or mechanical force per unit area exerted by a magnetic field upon a body is expressed as follows:

$$4.\quad F = \frac{\mu H^2}{8\pi}.$$

and, since air is the media used here,

$$5.\quad F = \frac{(\mu - 1)\, H^2}{8\pi}.$$

From equation 3, $\mu - 1 = 4\pi\chi$, and the equation reduces to

$$6.\quad F = \tfrac{1}{2}\chi H^2.$$

When one arm of a bent U-tube containing a magnetic liquid is introduced into the field, the liquid will be drawn into the field until the weight of the liquid held by the field balances the drawing force of the field, and the force in dynes will be equal to the hydrostatic pressure,

$$7.\quad h\sigma 980 = \tfrac{1}{2}\chi H^2.$$

From this equation either χ or H can be found when the other quantities are known. Since the salts were used in solution,

Table I.

Determinations of the Susceptibility of Water.

Date	Investigator	Temp.	Value
1903	Stearns	22°	-7.06×10^{-7}
1904	Wills	22°	-6.95
1912	Seve	24°	-7.2
1912	de Haas, Draper	24°	-7.3
1912	Weiss, Piccard	20°	-7.193
1913	Hayes, H.C.	24°	-7.26

allowance must be made for the water present and its susceptibility. The latter from Table I was taken to be -7.2×10^{-7}

and if need be the value which is given for volume could be reduced to that for mass by dividing by the density of water at the temperature used. As the correction is so small it was not used here. To obtain the susceptibility of the solution as a whole, the per cent of the anhydrous salt multiplied by its susceptibility is added to the per cent of water multiplied by its susceptibility. Then

$$8. \quad h\sigma 980 = \tfrac{1}{2}\chi_{sol.}H^2.$$

and

$$9. \quad H = \sqrt{\frac{2h\sigma 980}{\chi_{sol.}}}.$$

where $\chi_{sol.}$ is the magnetic susceptibility of the solution. When two liquids are used as a combination with different densities and susceptibilities the equation should be modified to the following form:

$$10. \quad \begin{cases} \sigma_s \times \dfrac{h}{\sigma_s/\sigma_s - \sigma_o} \times 980 = \tfrac{1}{2}(\chi_s - \chi_o) H^2 \\ h(\sigma_s - \sigma_o)\ 980 = \tfrac{1}{2}(\chi_s - \chi_o) H^2 \end{cases}$$

where σ_s and σ_o are the densities of the magnetic and less magnetic liquids respectively, and χ_s and χ_o are the susceptibilities. Thus the distance between the levels in the two arms is increased by a factor equal to the ratio of the density of the magnetic solution to the difference of the densities of the two solutions. Then

$$11. \quad H^2 = \frac{2h(\sigma_s - \sigma_o)\ 980}{(\chi_s - \chi_o)}$$

Previous Methods of Determining H and χ. The value of the experimental results in many investigations depends upon an accurate determination of field strengths, and in looking over experimental methods, it was found that investigators, with the exception of A.P. Wills, had been limited to the use of exploring coils for such work. From its galvanometer throw when the coil is put into or taken out of the field, its effective area, and the galvanometer constant, the field strength can be determined as follows:

$$H = 10^8 Rk\theta_1 / A ,$$

where R is the resistance in ohms, k is the galvanometer constant, θ_1 is the galvanometer throw and A is the effective area.

Quincke (3) used an exploring coil, calibrating the galvanometer from comparison with the current from an earth inductor which was rotated in a position where the earth's horizontal field had been carefully determined. J.A. Fleming and James Dewar (4) also used an exploring coil. Herman D. Stearns (5) used the coil method and determined its reliability by means of a current inductor which was standardized by means of an earth inductor. H.C. Hayes (6) used a coil but placed it in a stationary position, rotating the field past the coil. J. Jager and St. Meyer (7) used several methods, one of which was the bismuth coil which possesses the property of changing

resistance with change of magnetization. It is a delicate instrument and depends upon previous calibration for accuracy. They also compared the experimental field with one that was produced with known constants of construction and hence calculable.

In working out the magnetic susceptibility of water, A.P. Wills (8) used an original method for determination of field strengths, in which the pull of the field on a conductor carrying a known current is measured in grams of force, by means of balances. A slab of plaster of paris 8 x 4.5 x .5 cm. was arranged with a tinfoil conductor on the side and lower edges. In order to compute the field it was necessary to know the length of the conductor on the lower edge and this could be determined with exactness because of the sharp edges of the slab. The upper part of the slab was attached to one arm of a pair of balances and, by adjusting the balances before and after closing the circuit, the pull on the conductor was found. Then the field strength was equal to $\frac{10"g" F}{lI}$ where F was the pull in grams, l the length of the conductor and I the current in amperes as measured by a calibrated instrument.

S.C. Laws (9) used a solenoid whose field could be calculated. This makes four methods; the exploring coil, the bismuth coil, the conductor, and the solenoid.

In finding the magnetic susceptibility of weakly magnetic substances there was a greater diversity of procedure. G. Quincke (3) was one of the earlier investigators along this line and he used two interesting methods, the first by means of a bubble and the second by means of a capillary tube. In the first method the poles of a magnet were placed one above the other so that the field was vertical and the upper pole was bored through to contain a tube so that an air bubble could be blown into a given liquid whose upper surface was in contact with the pole. The upper end of the tube was connected to a carbon-bisulphid manometer. When the magnet was excited, the surface of the manometer fluid rose, if the liquid was magnetic, and fell, if it was diamagnetic. From the amount of rise or fall and the density of the CS_2 the force of the field could be calculated in grams. This was found by Quincke to be a troublesome manner of working and one that was limited to solutions that would not attack iron and were only weakly magnetic, in order not to change the field from the measured value. The manometer method has been ingeniously modified by O. Liebknecht and A.P Wills (10) for two purposes, one for studying the atomic susceptibilities of liquids and the other to study the drops of the liquids emitted under the control of the observer.

In Quincke's other method a tube bent twice at right angles was partly filled with the solution that was to be in-

vestigated and one arm was placed in the field while the other remained in the field that was almost null. The amount of rise or fall of the meniscus as the current was turned on or off, multiplied by the density, gave the pull of the field in grams and the susceptibility was calculated as above. This latter is the method commonly referred to as the Quincke Method.

Another procedure known as the Wills Method (8) is widely used. It consists of measuring the pull exerted by the field on a small vessel of the solution which was hung in the field and was large enough to extend to the null field outside. The vessels that he used were 8 x 4.5 x .5 cm. in size and were suspended from the balances so that the long edge was vertical and the broadest face was perpendicular to the field. Stearns (5), also Jager and Meyer (7) used the same method. H.C. Hayes (6) used a torsion method. A pair of delicate balances were suspended in a vacuum by a quartz fiber. In the place of pans were quartz standards on one of which was placed the glass capsule of water to be investigated. The force by which it was displaced as a horizontal field was moved across it, was measured by watching the deflection of a cross hair attached to one extended arm. The value of the deflection was computed from the torsion constant of the quartz fiber by which the balances were suspended, and the susceptibility was obtained from the following equation:

$$F_s = \chi MH \frac{dH}{dS}$$

where M is the mass.

J.A. Fleming and James Dewar (11) found that in working with liquid oxygen some method had to be found by which the results obtained would be independent of the mass, since the liquid oxygen evaporated very rapidly even in a good vacuum vessel, and independent of change of shape and conducting power of conductors immersed in it due to change of temperature. They decided upon the method of balancing two mutual inductances. A small wire transformer which was made upon a circular core of rectangular cross section was placed first in the liquid oxygen and the balance was adjusted with an induction coil; then lifted out into gaseous oxygen of about the same temperature, when the reading of the galvanometer was again taken. The permeability is

$$\mu = \frac{D}{A\,d/a} + 1 .$$

where D is the deflection when the current A is reversed, d is the deflection when calibrating current is used in the transformer alone, while A and a are the current strengths respectively.

Their second method was to find the apparent susceptibilities of a test ball in two different media, as air and liquid oxygen. The difference in the values gave the susceptibility of the liquid oxygen. Four test balls were used:

FIGURE I.

MAGNET CIRCUIT.

m---------magnet
Am 2------ameter
sc---------storage cells
R4,R3,----resistances/.

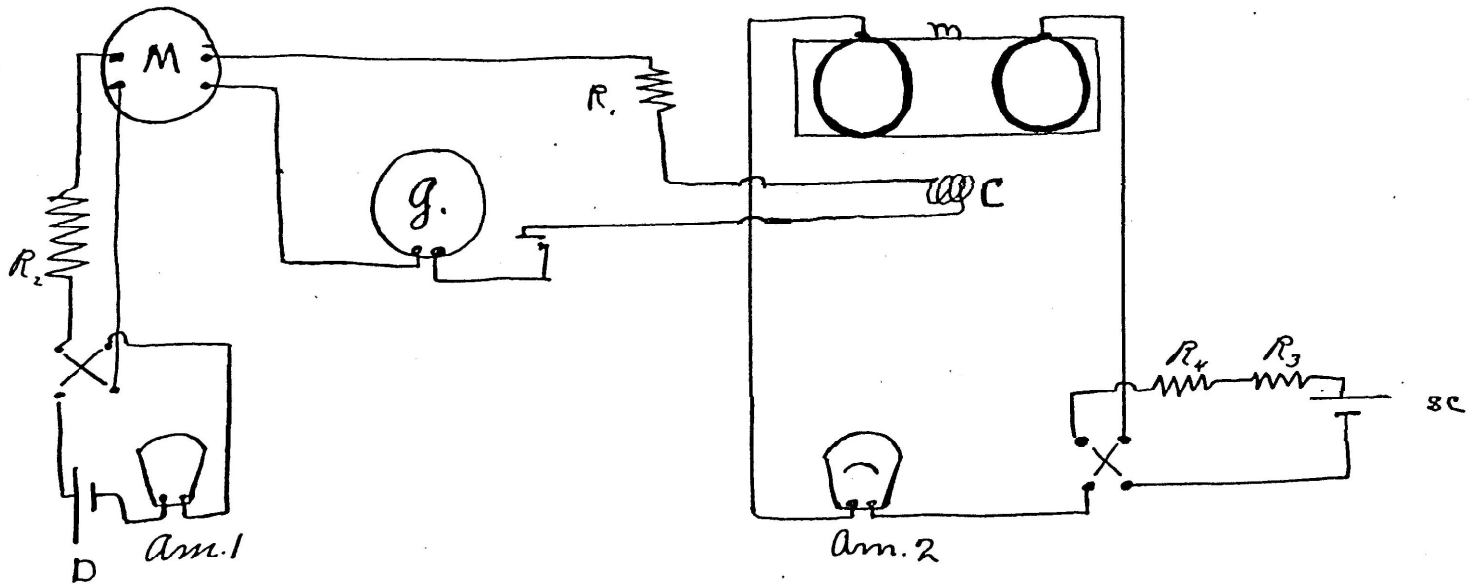
GALVANOMETER CIRCUIT.

G---------galvanometer.
RI,-------resistance.
M---------mutual inductance.
C---------exploring coil.

CALIBRATION CIRCUIT.

M---------mutual inductance.
R2,0------resistance.
Am I------ameter.
D---------dry cell.

FIGURE I.



silver, copper, glass, and bismuth, and they were hung from one arm of a pair of balances so that the force was measured in grams.

J.S. Townshend used a method very much like the last. He used an induction balance with a commutator that made contact about 16 times a second, and found after obtaining a balance that the insertion of a vessel of liquid in one of the coils would give a deflection equal to $4\pi\chi M$ where χ is the susceptibility and M, the mutual inductance. This value is equal to m the amount that is needed to change the galvanometer reading to zero. From this the susceptibility is equal to:

$$\chi = \frac{m}{4\pi M}.$$

Thus the examination of susceptibilities is principally done in one of six ways: the manometer method, the quincke method, the Wills method, the torsion method, the comparison method, and the induction-balance method.

Apparatus. The greater part of the apparatus including all the pieces that required a position undisturbed by jarring was placed on a large stone pier that was built upon bed rock and the sketch of connections is given in Fig. I. The magnet was placed on one edge so that the galvanometer could be as far as possible from the influence of the magnetic field. It was designed to produce a horizontal field and consisted of four coils of wire on two iron cores 5½ cm. in

diameter. The pole pieces used were elongated pieces of iron with tapered inside ends. Circular pole pieces with flat faces of 2.54 cm. diameter fitted closely into holes bored through the larger pieces. Any change of position of the smaller and larger pole pieces was prevented by a brass holder. Two spacings of the pole pieces were used: .58 cm. and .85 cm. The distances were obtained by the use of two flat brass disks of these thicknesses. The magnet was excited by a current from 30 storage cells and in series with it were a carbon variable resistance, a slide wire resistance, and a Weston ammeter No. 2298.

The field strength was obtained by means of a small exploring coil. Out of five coils, three were tested out as is shown by Table II, and No. I was chosen as the most reliable.

Table II.

Comparison of the Exploring Coils.

Coil	Pole Dist.	I_m	I_p	$G\theta_p$	$G\theta_H$	H
I	.58	8.±.1	.025	17.17	17.20 17.25	9456.15
II	.58	8.±.05	.0275	19	19.12 19.06	9476.65
III	.58	8.±.1	.0277	19.07	18.62 18.68	9339.57
I	.85	9.5± .1	.0195	13.37	13.90 13.86 13.86	7630.56
II	.85	9.5±.3	.022	15.19	15.38 15.46 15.35	7644.93
III	.85	9.5±.3	.0221	15.17	14.90 14.89 14.90	7626.59

The three were wound with No. 36 copper wire doubly insulated with silk. For No. I and No. II the wire was paraffined before winding and for NO. III shellacked. The constants of the coils are given in Table III.

Table III.

Constants of the Exploring Coils.

Coil	Material of Coil	Diameter of Coil	Turns	Diameter of Wire	Effective area A.
I	hard rubber	1.349	9	.0205	13.25
II	" "	1.342	10	.0205	14.578
III	brass ring	1.395	9	.021	14.173

The coils were mounted on a steady wooden mounting which could be moved in a wooden track so that, in testing the coils, they could be placed in the field in succession. They were pulled into and out of the field by pieces of fish line tied to corks so that the apparatus during the manipulation of the coils was touched as little as possible.

In series with the exploring coil were these pieces: a resistance, the secondary of a mutual induction coil of 50 millihenry's inductance, Leeds Northup and Co. No. 16089, the galvanometer and key. The galvanometer was a Leeds, Northup and Co. "Type P" ballistic galvanometer No. 4936. During the preliminary readings the galvanometer had a sensitiveness of .022 x 10^{-5} for I, but before taking the readings with the closed containers the fiber was changed for a finer one obtaining a sensitiveness of .008 x 10^{-5}. It was standardized for each field strength by means of a dry cell, milli-

amperemeter, resistance, and mutual inductance that was in series with the galvanometer. The milli-amperemeter was a Siemens and Halske instrument in good condition. A damping key short circuited the galvanometer. The deflections were read by means of a telescope and scale placed about a meter away.

In the preliminary tests the solutions were used in glass tubes of small bore which were bent twice at right angles so that one arm could be placed vertically in a horizontal magnetic field, while the other was supported by a wooden clamp 20 to 30 cm. from the pole pieces of the magnet.

For the final readings three continuous containers which Dr. Kester very kindly made were used. They consisted of a tube of small bore for the field arm, of a bulb of about 6 cm. diameter with larger tubing for the distant arm, and of the connecting tubing of the larger bore. The upper connection had two entrance tubes for filling. The three are shown in Plate I-A.

Since it was desirable to use the values as previously determined for the susceptibility of substances used, the containers and tubes were carefully cleaned. They were washed with potassium hydroxide and acid cleaning solutions, and rinsed several times with distilled water. The tubes for preliminary trials were not dried but were washed once or twice with the solution to be used. For the later read-

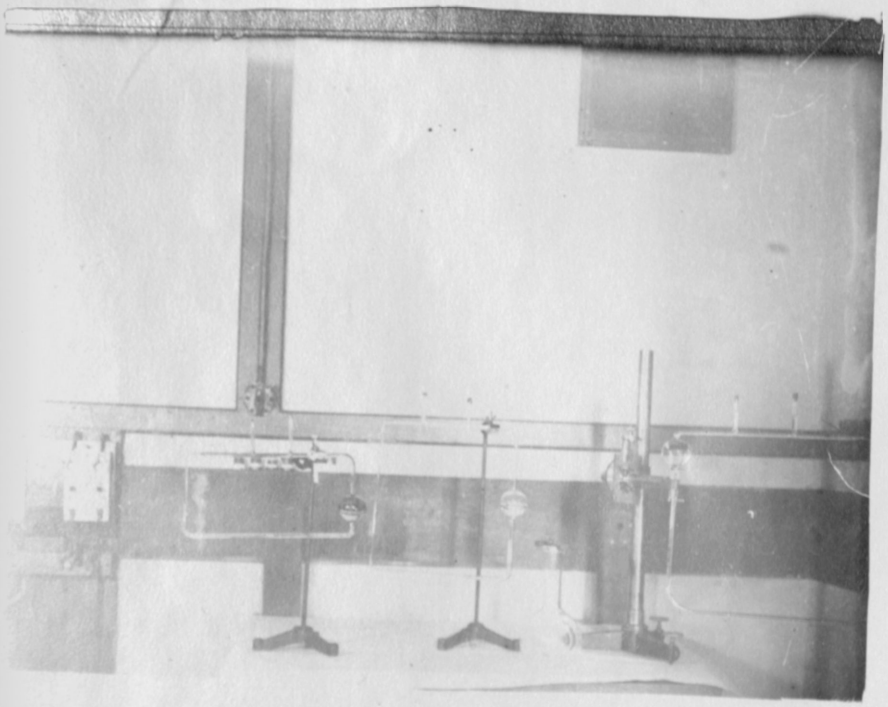


PLATE I-A

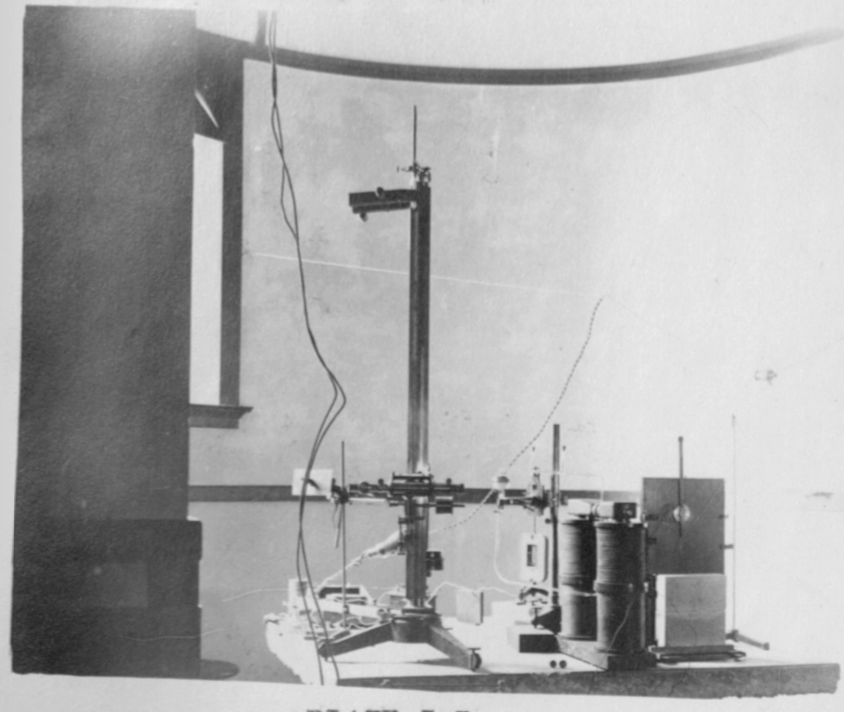


PLATE I-B

ings, the containers were washed with absolute alcahol and dried by means of an aspirator that drew air through the container from a calcium chloride tube. The same precautions were taken in cleaning the cup and pendant of the Mohr-Westphal balances and any beaker or funnel used in handling the solutions.

The support for the bulb was made by using the brass standard and telescope holder of a Geneva Society cathetometer 66 cm. high. A brass clamp and strap were fitted to a heavy brass rod that was turned to fit the telescope tube. The brass rod which was fitted into the telescope tube was long enough to permit the container to be placed at right angles to it while the standard was placed in line with the two poles. This arrangement made it possible to swing the container in and out of the field in a straight line for a few centimeters with little jarring. The whole carriage could be moved up and down the standard or could be clamped and moved a few centimeters by the micrometer screw which could be adjusted to 1/400 of a millimeter. The support is shown at the right of Plate I-A.

A large Geneva Society cathetometer 140 cm. in height, was used to view the rise of the liquid in the field. It was all made of brass with the exception of the base which was of iron and the narrow steel scale that extended the

length of the shaft. The latter turned on a core and could be leveled by screws according to the two levels attached to its base. The telescope carriage could be raised or lowered on the shaft or fastened and adjusted by means of a micrometer screw. The vernier read to 1/50 of a millimeter. The weight of the carriage was partially balanced by weights and pulleys. The cathetometer was placed on a pier to assure more steadiness in the level of the telescope as it was found that walking about the room produced jars sufficient to change the level. This is shown with the rest of the apparatus in Plate I—B.

The Mohr-Westphal balance was used to determine the density of the liquids, and the values were checked by using the pycknometer. The weights used with the balance were carefully weighed, using weights that had been standardized. The error due to the weights could not exceed 1/10 of 1%. The relation of the weights as used on the balance was checked out with distilled water and found to be correct.

A short magnet 4.3 cm. long suspended by an untwisted fiber in a glass flask was used to find the value of the field about the magnet where the exploring coil stood and where the meniscus stood when the field was off. Comparing the vibration periods of the magnet in these positions with its period in the Electricity Laboratory at a position where the value of the earth's field was .21 shows that those fields can be

taken as null fields. The results are given in Table IV.

Table IV.

Value of Null Fields.

Position	Time	Vib.	1/2 Period	H
Bulb with no field	60 sec.	80	1.38	.461
	50.4 "	80		
	41.4 "	30	1.38	
Bulb with field	60 "	100	.3	2.106
	60.4 "	100	.302	
	30.4 "	50	.304	
	30.4 "	50	.303	
Electricity Lab.	125.6	40	3.14	.21
	61.4	20	3.07	
	27.4	10	2.74	
	60.8	20	3.04	
	31.2	10	3.12	
Coil out of field	32.8	100	.328	.19
	46.4	140	.331	
	19.8	60	.330	
Meniscus field off	22.5	20	1.12	.59

Much difficulty was experienced with the galvanometer, as the surface currents due to the damp season seemed to have a ground. All the apparatus in the galvanometer circuit was placed on pieces of hard rubber that had been washed in distilled water, the heavy current was changed from under-floor to over-head wires and the resistances in the magnet circuit were placed on an insulated stool, but it was not until the magnet was placed on disks of melted sulphur that the leakage currents were absolutely stopped.

Method. The plan for conducting this work was to try out a number of solutions to find the one that gave the greatest height under the influence of the magnetic field and then to examine the result of combining this liquid with one that was emiscible with water to increase the height to which the meniscus rose.

A number of substances whose magnetic susceptibilities ran high as given in the Tabellen of Landolt, Börnstein, and Meyerhofer were made into solutions of varying densities with water as a solvent. Other solvents whose negative susceptibilities were lower than that of water were considered but the solubilities of the more magnetic salts were so much lower with the other solvents than with water that water was taken as the best. It was obtained from the water analysis laboratory where it was distilled so as to be free from ammonia, chlorine, and other materials.

The following Table V shows roughly that with the low value of susceptibility for water the effect of a magnetic field varies as the concentration of the solution. In this and the following tables χ is the magnetic susceptibility, ρ is the density, $G\theta_n$ is the galvanometer throw for field strength, H, and h is the rise of the liquid upon applying the field. The galvanometer was calibrated for each reading. The residual field was found to be very constant.

Table V.

No.	Subs.	χ	σ	$g\theta_H$	h in cm.
III	$MnCl_2$	117	1.079	2.24	.0793
	"	to		2.22	.0677
	"	122		2.21	.0705
	"		"	2.22	.0668
V	"		1.125	2.2	.127
	"		"	2.25	.102
IV	"		1.258	2.3	.247
I	Fe_2Cl_6	72.5	1.065	2.2	.419
	"	to	"	2.2	.306
	"	91.8	"	2.23	.52
II	"		1.178	2.2	1.29
	"		"	2.2	1.28
VIII	"		Con	2.2	2.21
	"		"	2.2	2.094
	"		"	2.26	2.080
	"		"	2.25	2.11

Table VI gives a few test readings that can be compared from the galvanometer throws as the constant varied between the limits of 17.4 and 17.33.

Table VI.

I	$G\theta_H$	$G\theta_{H_0}$	Time
8.2	2.2	.13	
"	2.2	.13	Immediately after.
"		.13	5 minutes later.
10	2.26	.13	"
9.9	2.25	.13	"
9.75	2.28	.13	"
		.13	24 hrs. later.

From Table VII can be seen the relative values of the concentrated solutions of several materials when placed in the same field strength.

Table VII

Sub.			G	h 0m.
Fe_2Cl_6	72.5 to 91.8	Con	2.27	.208
$Mn\ SO_4$	114 to 98.4	"	2.24	.130
$Mn\ Cl_2$	117 to 127	"	2.24	.39
$Co\ Cl_2$	81 to 101	"	2.25	.235

The $Mn\ So_4$ gave the lowest "h" because of its low solubility while $Co\ Cl_2$ came second. $Mn\ Cl_2$ gave the best results and was used for the remaining work.

To increase the actual rise that was viewed by means of the cathetometer, the arm that was in the null field contained a bulb of about 6 cm. diameter so that with the liquid level at the widest diameter, the total difference in level of the liquids in the two arms was the column viewed through the cathetometer.

Professor Cady suggested the use of two liquids in a continuous loop of glass tubing and, upon his advice, chloroform and carbon tetra chloride were tried out with $Mn\ Cl_2$. Chloroform had a density of 1.47 which was close to that of the $Mn\ Cl_2$ solution which was 1.459, and there could be expected a column seventy times as high as with the $Mn\ Cl_2$ alone. But associated with this advantage is the disadvantage of the slow movement of the mass and the breaking off of globules of one substance into the other. The two formed

very peculiar surface tension effects that caused the liquids to cling to the walls of a narrow test tube in curious shapes, and, because of this action a definite meniscus separating the two could not be obtained.

With CCl_4 the action was better and, with careful handling, a clear meniscus existed between the two. The CCl_4 had a density of 1.59 and took the lower position in the tube, so that the $Mn\ Cl_2$ was drawn downward when in the field.

The field was measured before placing the field arm of the container in it. Then the carriage which held the tube was raised slowly, until the meniscus was near the center of the field and then fastened. The final raising of the carriage was accomplished with a micrometer screw and it was turned by small amounts as the $Mn\ Cl_2$ was drawn downward into the field. When a steady position was reached the field was turned off and the meniscus rose in the tube. The error arising from neglecting the fall and rise in the bulb was less than 1/3 of one per cent so that readings were taken only with the field arm.

As a continuous glass container is difficult to make and rather frail to use, a piece consisting of several parts fitted together with rubber tubing would be more convenient. $C\ Cl_4$ dissolves rubber and on that account would not be suitable. Olive oil could be used and is better than kerosene, because of greater density, .91 in comparison with .81 for kerosene. Although in time olive oil would disintegrate the rubber for reasonable periods it could safely be used, as

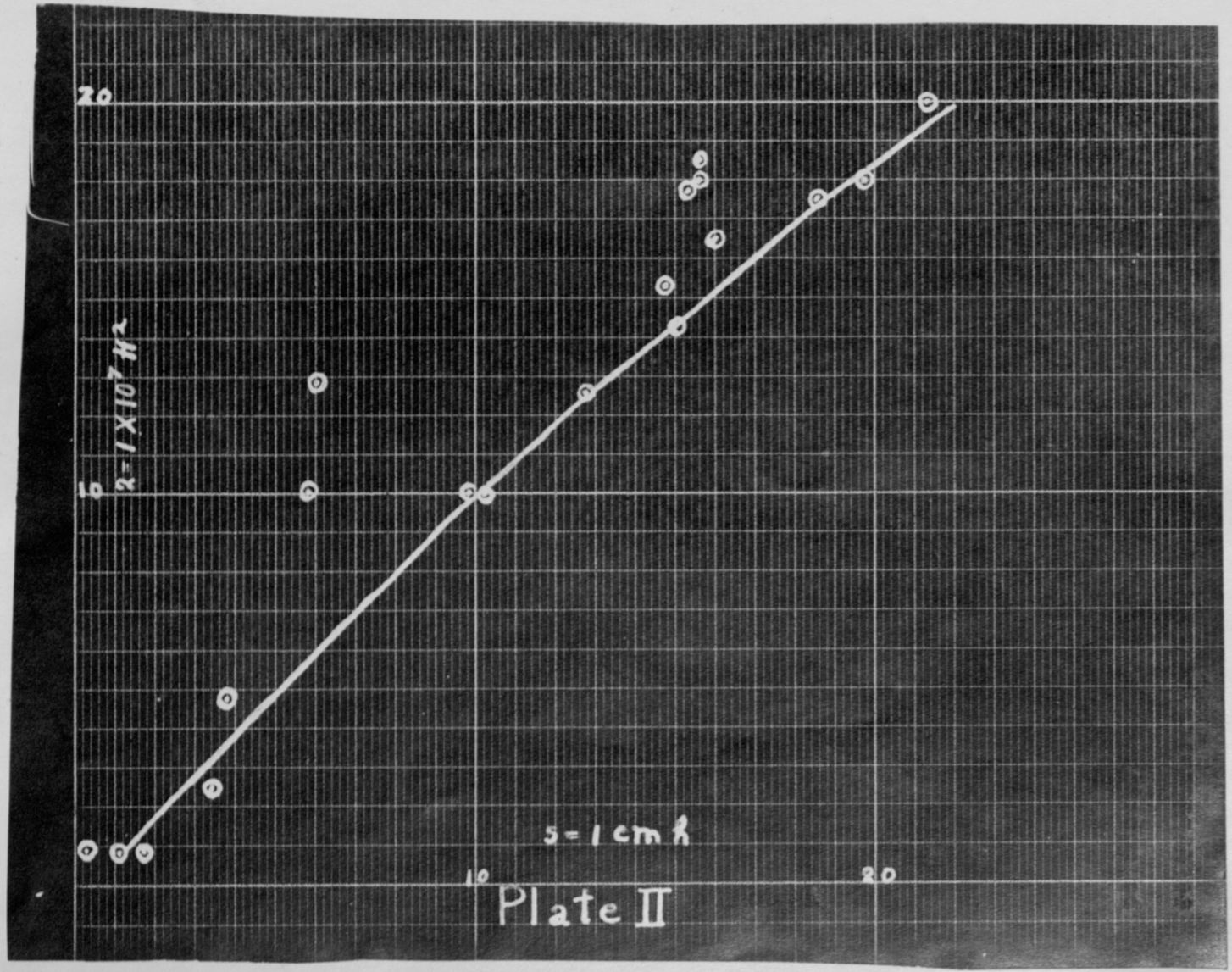


Plate II

little of the rubber need be exposed at the joints and when any doubt should be felt as to the condition of the oil, due to dissolving some substance from the rubber, fresh oil could be substituted.

Results. Olive oil and Mn Cl_2 worked nicely together having a clear meniscus between the two liquids and having a difference in density sufficient to give appreciable readings. Plate II shows a curve for "h" with corresponding H^2. The headings given in Table VIII were taken with the magnetizing current and the height "h" so that it was necessary to select an average field value for the given current. This makes a small error which would not change the first two figures in the square of the field.

Table VIII.

I	"h"	I	"h"	I	"h"
9.	3.136	.4	.060	8.	3.958
8.	3.120	.4	.350	7.	3.688
7.	3.064	.4	.180	6.	3.458
5.	2.942	1.	.764	4.	2.994
3.	2.542	2.	1.160	2.	1.960
2.	2.040	3.	1.236		
0.7	.680	10.	4.238		

The Mn Cl_2 and C Cl_4 came to rest under the action of the field in about ten minutes and it was watched a minute or two to see if the position were a final one. When the magnetizing current was opened the meniscus retreated instantly to within a centimeter or two at its final position. Table IX gives the results which show that after ten minutes

TABLE. IX. $MnCl_2$ and CCl_4

No.	I_p	Gal θ_p R.	$\mathcal{L}$.	Gal θ_m R.	$\mathcal{L}$.	I_m	I-8m	I-5m
I	.0084	I6.80	I6.80	I7.65	I7.70	I0.4	56.7I6	
2	.0084	I6.I0	I6.I0	I7.00	I7.I0	9.7	36.860	
3	.0084	I6.20	I5.99	I7.07	I6.98	9.8	57.0I5	
4	.0I9I	I2.90	I2.76	I2.75	I2.76	2.0	24.240	24.300
5	.0I92	I2.92	I2.80	I2.55	I2.55	2.0	24.I88	24.282
6	.0I92	I2.87	I2.68	I2.54	I2.45	2.0	23.056	23.I40
7	.0I85	I2.40	I2.22	I2.56	I2.47	2.0	23.400	23.494
8	.0I46	9.80	9.70	9.I2	9.I3	I.0	I7.556	I7.6I8
9	.0I3I	8.58	8.78	8.85	8.78	I.0	I7.I22	I7.2I0
I0	.0I5I	8.88	8.78	8.47	8.40	I.0	I6.790	I6.850
II	.0I28	8.44	8.37	8.45	8.4I	I.0	I6.7I2	I6.7I6
I2	.0I22	8.25	8.I8	8.30	8.25	I.0	I6.456	I6.574
I3	.02II	I4.54	I4.27	I4.30	I4.20	3.0	27.448	27.550
I4	.02II	I4.30	I4.I3	I4.20	I4.I2	3.0	27.695	27.742
I5	.02I2	I4.37	I4.I8	I4.2I	I4.I5	3.0	28.I40	28.282
I6	.0229	I5.I8	I5.0I	I4.92	I4.84	4.0	30.770	30.796
I7	.0229	I5.2I	I5.0I	I4.96	I4.86	4.0	30.244	30.330
I8	.0229	I5.33	I5.03	I4.97	I4.85	4.0	30.566	30.728
I9	.023I	I5.45	I5.25	I5.57	I5.44	5.0	3I.772	3I.833
20	.023I	I5.49	I5.55	I5.52	I5.70	5.0	3I.862	3I.958
2I	.0233	I5.58	I5.30	I5.00	I5.09	5.0	3I.966	32.I80
22	.0245	I6.40	I6.35	I6.02	I6.02	6.0	33.028	33.I46
23	.0244	I6.58	I6.47	I6.00	I6.04	6.0	33.280	33.352
24	.0242	I6.47	I6.38	I5.99	I6.05	6.0	33.486	33.500
25	.0236	I4.30	I4.59	I4.72	I5.I0	6.0	3I.828	3I.880
26	.0253	I5.50	I5.55	I5.00	I5.34	7.0	32.460	32.488
27	.0253	I5.29	I5.5I	I5.00	I5.33	7.0	32.642	32.672
28	.0253	I5.3I	I5.5I	I5.36	I5.60	8.0	33.I28	33.I32
29	.0253	I5.30	I5.50	I5.23	I5.5I	8.0		33.260
30	.026I	I5.8I	I6.03	I5.44	I5.7I	9.0	33.960	33.978
3I	.026I	I6.00	I5.88	I5.75	I5.46	9.0	34.340	34.370
32	.0068	3.60	3.60	3.60	3.60	0.3	I4.382	I4.386
33	.0025	I.60	I.6I	I.25	I.20	Residual Field		
34	.0062	3.80	3.82	3.75	3.80	0.4	I4.388	I4.388
35	.0085	5.20	5.I9	5.23	5.30	0.6	I4.434	I4.434
36	.0029	I.83	I.82	I.20	I.I9	Residual Field		
37	.0084	5.83	5.84	6.9I	6.98	0.8	I4.5I0	I4.5I0
38	.0020	I.27	I.37	I.I7	I.I7	Residual Field		
39	.027	I6.I9	I6.38	I5.79	I5.97	II.	34.968	34.098
40	.0259	I5.60	I5.72	I5.6I	I5.82	I0.	35.290	35.3I4
4I	.0II6	6.98	6.89	7.37	7.44	0.9	I4.874	I4.9I0
42	.0I89	II.2I	II.26	I0.45	I0.53	I.5	20.280	20.3I4
43	.0I74	I0.45	I0.50	I0.I2	I0.20	I.5	I8.978	I9I040
44	.02I2	I2.68	I2.80	I2.50	I2.60	2.5	25.I40	25.208
45	.026	I5.39	I5.60	I5.79	I5.95	II.	35.578	

TABLE IX. $MnCl_2$ and CCl_4

No	L-10m	L- m	T C	h	H	C	$H^2 - H_0^2$
I			25°.4	22.370	9529	II770	9.079×10^7
2				22.5I4	9585	II820	9.I89
3	37.360	37.282	25°.2	22.670	9575	II870	9.I67
4	24.380	24.380	23°.8	9.894	7I48	8797	5.09I
5	24.488	24.496		9.842	7067	8775	4.993
6	23.2I5	23.244		8.7I2	7078	7357	5.007
7	23.530			9.054	7099	7499	5.04
8	I7.742		24°.I	3.2I0	5I58	4466	2.66
9	I7.330			2.776	4942	4I53	2.44
I0	I6.950	I7.206		2.444	4723	3898	2.23
II	I6.844			2.366	4840	3833	2.34
I2	I6.760	I7.054	24°.8	2.II0	4635	362I	2.I4
I3	27.734		25°.7	I3.I02	7884	9020	3.2I
I4	27.64			I3.339	7926	9I03	6.28
I5	28.420			I3.794	7940	9257	6.30
I6	30.832		25°.7	I6.422	85I4	I0I00	7.25
I7	30.484		25°.5	I5.898	8504	9938	7.23
I8	30.822			I6.320	85I4	I0070	7.24
I9	3I.9I6			I7.426	880I	I0400	7.75
20	32.I44		25°.2	I7.5I6	8765	I0430	7.68
2I	32.2I6			I7.620	8743	I0450	7.64
22	33,250	33.486	25°.4	I8.682	9045	I0770	8.I8
23	33.442	33.970		I8.930	8925	I0840	7.99
24	33.5I2	33.844		I9.I40	885I	I0900	7.83
25	3I.954		20°.6	I7.482	9I49	I0420	8.37
26	32.538			I8.II4	9385	I0600	8.8I
27	32.7I2			I8.296	9395	I0660	8.83
28	33.I50		2I°.6	I8.782	9585	I0800	9.20
29	33.300				9528		9.07
30	34.080		2I°.6	I8.6I4	96I5	I0730	9.24
3I	34.420			I9.994	9663	I2400	9.34
32	I4.386		22°.4	.036	2I92	473	0.446
33					572		
34	I4.388			.042	2320	573	0.505
35				.088	3249	739	I.00
36					7I7		
37			22°.7	.I64	3772	I0I0	I.37
38					668		
39	35.I66	35.256		20.622	9960	I2750	9.92
40	35.576			20.944	9809	II400	9.62
4I				.528	466I	I8I0	2.I7
42				5.934	6639	6070	4.4I
43		I0.206		4.632	6348	5364	4.03
44				I0.794	7676	8I88	6.20
45	35.620			2I.232	I0000	II480	I0.I0

the change was small. To obtain the complete change required a wait of one or two hours. As it was impossible to spend so much time for each reading, the position was taken at three, five, and ten minutes, and at longer intervals as there was occasion to leave the apparatus.

In the Table IX I_p was the current in the primary of the induction coil for calibrating the galvanometer. Gal θ_p, R and L, are the right and left throws of the galvanometer for calibration as the current in the primary was made and broken. Gal $\theta_\#$ R and L are the right and left throws when the exploring coil was used. I_m was the current strength in the magnet circuit. L-3 min., L-5 min., and L-10 min. are the positions read off on the cathetometer at these times. T was the temperature but no corrections were made as they would be small. "h" was the height or distance between the levels when the field was excited and taken off, and was found by subtracting the position of the cathetometer when focused on the center of the field 14.346 cm., from its position when focused on the meniscus three minutes after taking off the field; H was the field strength obtained from the following equation:

$$H = \frac{M 10^8 \; I \; \theta_2}{A \; \theta_1}$$

which was obtained from these two

$$k\theta_2 = \frac{MI}{R}, \quad \text{and} \quad k\theta_1 = \frac{AH}{10^8 R},$$

where H is the field; M the mutual inductance; I the current

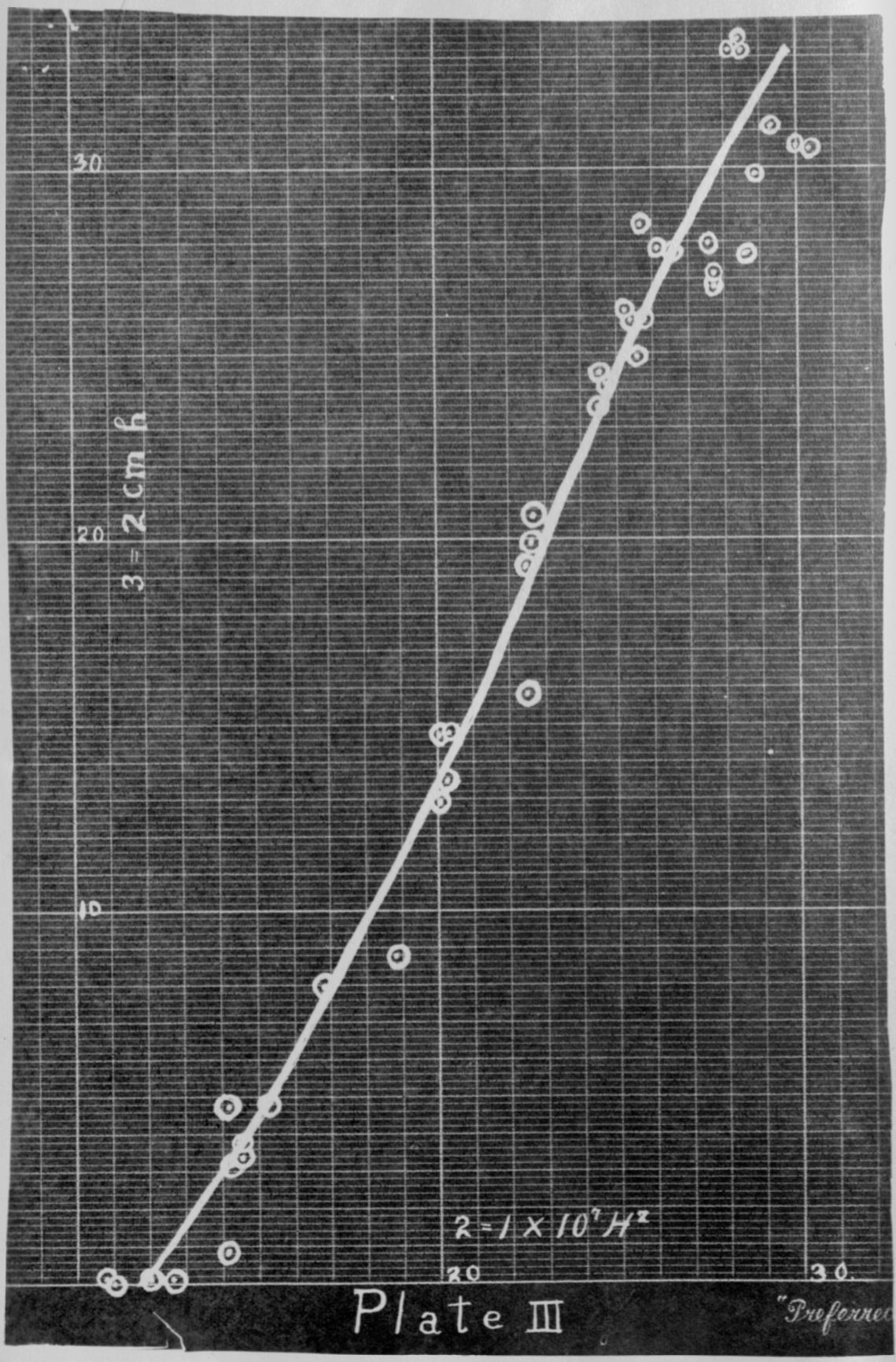


Plate III

"Preferred

in the primary circuit; θ_2 the galvanometer throw when the current in the primary circuit was made or broken; k the constant of the galvanometer; θ_1 the throw when the exploring coil was used; and A the effective area of the coil. For the last column all readings excepting the 32, 34, 35, and 37, the field where meniscus stood after turning off the field was counted as null, but for the named readings the meniscus stood between the poles and the residual field was taken.

Plate III shows the curve which was plotted from the "h" for three minutes and the corresponding square of the field.

Plate IV shows the curve for the magnetic susceptibility of $C Cl_4$ which was plotted from Table X, where the Headings of the columns are the same as occurred in the previous table, excepting χ which stands for the magnetic susceptibility of the $C Cl_4$. Leaving out NO. 6 and 20, which seem to be exceptions, the average for susceptibility is $+.81 \times 10$ which is close to that of Beryellium .79, magnesium .57, and copper oxide .73. The organic liquids as listed in the Tabellen are all negative.

To check the use of $Mn Cl_2$ with $C Cl_4$ for determination of field strength it was necessary to know the amount of weight

TABLE X. For the Susceptibility of CCl_4.

No.	I_p	Gal θ_p R	$\mathcal{L}$.	Gal θ_H R.	$\mathcal{L}$.	I_m.	L-3m	T-C.	H	H^2	x
I	.025	I5.55	I5.32	I6.87	I5.90	I0.95	I4.370	22°.I	.028	9.54xI0^7	.89xI0^{-6}
2	.025	I5.38	I5.49	I5.83	I5.92	II.00	I4.370		.024	9.4I	.795
3	..025	I5.35	I5.49	I5.85	I5.86	II.00	I4.362		.0I6	9.4I	.53
4	.025	I5.49	I5.5I	I5.85	I5.87	II.00	I4.378	24°.0	.032	9,32	I.07
5	..025	I5.47	I5.57	I5.84	I5.79	II.00	I4.374		.028	9.24	.944
6	.025	I5.50	I5.52	I5.92	I5.85	II.50	I4.4I4		.068	9.34	2.26
7	.025	I5.47	I5.5I	I5.70	I5.70	I0.00	I4.356		.0I0	9.I4	..34
8	.025	I5.49	I5.40	I5.7I	I5.67	I0.00	I4.376		.030	9.I7	I.02
9	.025	I5.50	I5.42	I5.55	I5.48	9.00	I4.366		.020	8.96	.69
I0	.025	I5.40	I5.50	I5.52	I5.55	9.00	I4.374		.028	8.92	.98
II	.025	I5.49	I5.39	I5.30	I5.27	8.00	I4.376		.030	8.72	I.07
I2.	..025	I5.47	I5.40	I5.33	I5.35	8.00	I4.372		.026	8.77	.92
I3	.023	I4.43	I4.30	I5.I0	I5.07	7.00	I4.370	23°.7	.024	8.28	.90
I4.	.023I	I4.33	I4.33	I5.09	I5.05	7.00	I4.370		.024	8.38	.89
I5	.0232	I4.47	I4.37	I4.82	I4.77	6.00	I4.368		.022	8.07	.85
I6	.023I	I4.37	I4.37	I4.47	I4.40	5.00	I4.366	23°.8	.020	7.72	.8I
I7	.0222	I3.92	I3.80	I3.9I	I3.83	4.00	I4.360		.0I4	7.02	.62
I8	.02I3	I3.28	I3.I4	I3.30	I3.I8	3.00	I4.360	23°.6	.0I4	6.79	.64
I9	.0I9	II.80	II.72	II.80	II.69	2.00	I4.356		.0I0	5.I4	.60
20	.0I2	7.49	7.49	7.72	7.82	I.00	I4.352		.006	2.2I	.846
2I	.0075	4.68	4.64	4.43	4.46	..50	I4.350	23°.5	.004	.74	I.70
										MEAN----	.8I

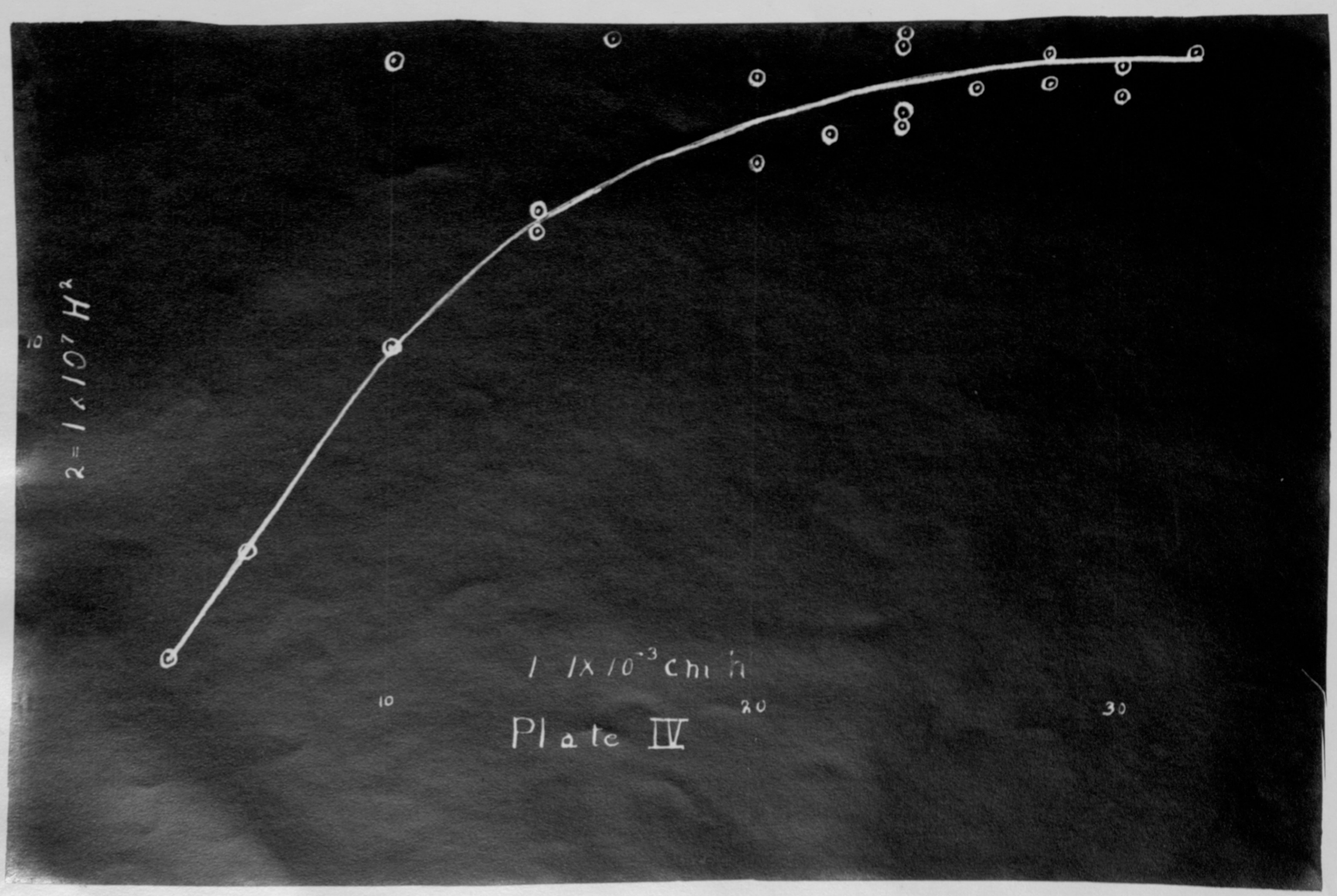


Plate IV

of the anhydrous salt in the solution and this was found by quantitative analysis in the following manner. 20 c. cm. of the solution was evaporated and dried at 100 C. The product was the salt with 2 moles of water and from this the amount of anhydrous salt was found to be 9.93 gms. or 34 per cent.

From the following equation

$$\chi_{sol} = p \times \chi_{an} + (1 - p)\chi_{w}$$

the magnetic susceptibility of the solution is obtained where χ_{sol} is the susceptibility of the solution, p is the per cent of anhydrous salt present, χ_{an} is the susceptibility of the latter, $(1-p)$ is the per cent of water, and χ_{w} is its susceptibility.

From this equation and equation 11, the field strength was calculated for the heights given in the h column of Table IX and the results are given in column $\mathfrak{C}$ of the same table.

The weakness of the method lies in the time which it takes for the liquids to assume a stable position after turning off the field. This reluctance is due to the nearness of density of the two materials and the use of a long tube for the purpose of reaching a null field.

Conclusion. From these results it seems possible to

construct a practical device for measuring field strengths by means of liquids; and it appears that CCl_4 is a magnetic liquid with a magnetic susceptibility of about $+.81 \times 10^{-6}$.

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Lawrence, Kansas.
July 26, 1915.